LATENT VALUE ESTIMATION IN CONTINGENT VALUATION

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Abstract

This paper explores the concept of a latent value of a commodity not normally traded in the market in the context of contingent valuation. The seller's willingness to accept compensation (WTA) is assumed to be a linear mix of unknown independent signals consisting of the latent value (P) of the commodity and an adjustment factor (F). Similarly, the buyer's willingness to pay (WTP) is assumed to be an independent mix of these two unobservable signals. The concepts of "willing to pay" (WTP) and "willing to accept" (WTA) are often used to value individuals' preferences and select regarding gains and losses. Valuing goods and services not normally traded in the market. The ICA platform as a means to estimate the latent price P in contingent valuation. The two unknown independent signals are estimated by Independent Component Analysis (ICA) and by Factor Analysis methods. Knowledge of the latent value and adjustment factor aids in the negotiation process in contingent valuation in environmental economics.

Keywords: contingent valuation, willingness to pay, willingness to accept compensation, latent value, factor analysis, independent component analysis

1.0 Introduction

Popular methods of valuing publicly provided goods and services require linkages to actual market transactions (Bishop (1990)). Goods and services not normally traded in the market were priced using contingent valuation methods. In this method, a seller sets a price for the good or service called his "willingness-to-accept" (WTA) compensation while a potential buyer declares his "willingness-to-pay" (WTP) for the good or service and a transaction occurs when WTA=WTP. Unlike classical pricing models in economics where prices are set based on production costs and profit, in contingent valuation, the goods or services are not "produced" in a strict sense, but are rather provided by nature, to both WTA and WTP reflect the personal appreciation of the seller and buyer of the value of the good or service. This is a popular problem of economists, on how to account for the non-produced goods and services by human effort. This paper explores and proposes an economic value for those goods and services that are not normally traded in the market. Although WTA and WTP are personal preferences of the economic players, there is a latent or inherent price reference that both players, consciously or unconsciously, refer to.

The concept of a latent or inherent value is pervasive in Statistics. Latent canonical correlation (Johnson and Wichern,2007), factor analysis (Anderson, (1988)), principal components analysis (Okamoto et al.,2012). Despite of the researches conducted, all speak of hidden values which are uncovered by specialized statistical methodologies. All latent variables techniques begin from the assumption that each observed random observation carries information about the "true value" of the variable plus noise. In independent component analysis (ICA), the observed value is assumed to carry information about independent signals which are linearly mixed (Comon, 1994). Hyvarinen (1997) sets the ICA model as:

1.1 $X=As$

where $X=(x\_1,x\_2,\ldots,x\_n)^T$, $A=(a\_ij)$ is an mxn matrix of constants and $S=(s\_1,s\_2,\ldots,s\_n)^T$ are independent (unknown) signals. An estimate of the matrix A is obtained by maximizing the independence of the signals S and S is recovered from:

1.2 $S=A^{-1}x=Wx.$

We make use of the ICA platform as a means to estimate the latent price P in contingent valuation.

2.0 Basic Concepts

We begin by defining the balanced ICA model:

Definition 1. Let $X=(x\_1,x\_2,\ldots,x\_n)^T$ be an n-dimensional observed random vector, let $A=\alpha\_ij$ be an n x n matrix of constant, $S=(s_1,s_2,\ldots,s_n)$ be mutually independent non-Gaussian signals which are not observed. Then, the balanced ICA model is defined as:

$$X=AS.$$

Using the independence of the unobserved signals, we wish to derive an estimate of A. When the signals $(s_1,s_2,\ldots,s_n)$ are in fact Gaussian, then zero correlation implies independence and conversely. However, when they are not Gaussian, zero correlation need not imply independence. We survey some of the alternative approaches in the latter case.

Maximum-Likelihood Method. Maximum likelihood estimators of parameters in statistical models are of interest because MLE's are asymptotically unbiased, efficient and the variance of the asymptotic distributions achieve the Cramer-Rao lower bound (Lehmann, 1984). Suppose that the probability density function of *s_i,i=1,2,…,n* is known of $q(s_i)$, then the joint pdf of $(s_1, s_2, \dots, s_n)$ can be written as:

$$2.1 \qquad q(s_1, \dots, s_n) = \prod_{i=1}^{n} q(S_i).$$

We seek the distribution of X=AS. This can be obtained by the usual transformation of variables technique:

$$2.2 \qquad p(X; A) = \frac{1}{|\det(A)|} \cdot \prod_{i=1}^{n} q(S_i).$$

Let $W=A^{-1}$, then the log-likelihood function can be written as:

$$2.3 \qquad L(X; A) = \log p(X; A) = log\ |\det(A)| + \prod_{i=1}^{n} \log q\ (w_i^T x).$$

We now want to find the matrix W that maximizes (2.3). If the pdf q(.) is of the form: $q(S_i) = e^{-h(s\_i)}$, then (2.3) simplifies to:

$$2.4 \qquad L(X; A) = \log \det(w) - \prod_{i=1}^{n} h\ (w_i^T x).$$

The technique of maximum likelihood estimation requires knowledge of the pdf $q(S_i)$. In most cases, $q(S_i)$ is unknown and $h(S_i) \neq -\log q(S_i)$. (Phan and Darrat (1992)) showed that for certain classes of function $h(S_i) \neq -\log q(S_i)$, the estimators (called quasi-likelihood estimators) remain asymptotically efficient.

When T samples of X are observable, the objective function (2.4) becomes:

$$(2.5) \quad L\left(X^{(T)}; A\right) = \det(w) - E_T\left(\sum_{i=1}^{n} h\ (w_i^T x_i)\right)$$

Information-Theoretic Method. Alternative estimation procedures are suggested in the literature (Hyvarinen (1997), Cardoso (1992), OJA and Hyvarinen (1996)) which utilize an information-theoretic approach instead of the maximum-likelihood approach.

Definition 2. The differential entropy or random vector Y with density f(.) is given by:

$$H(Y) = H(Y_{gauss}) - H(Y).$$

The negative normalized entropy or negentropy is:

$$J(Y)=H(Y_{gauss})-H(Y).$$

The negentropy of Y is a non-negative value and is zero if and only if Y has a Gaussian distribution. Maximizing J(Y) is, therefore, the same as maximizing the non-Gaussianity of the components of Y.

Using the concept of differential entropy, we define the mutual information I between n scalar random variables $y_1, y_2, \ldots, y_n$:

Definition 3. The mutual information $I(y_1, \ldots, y_n)$ of the scalar random variables y_i, $i=1,2,\ldots,n$ is:

$$I(y_1, y_2, \ldots, y_n) = \sum_{i=1}^{n} H(y_i) - H(Y)$$

If $y_1, y_2, \ldots, y_n$ are independent, then I=0 and conversely. Minimizing I is, therefore, equivalent to finding mutually independent component $y_1, y_2, \ldots, y_n$.

In the context of ICA, finding the transformation matrix W that minimizes mutual information is equivalent to finding direction in which the negentropy is maximized (Novey M. et al. (2008)). We can express I terms of the negentropy of Y as:

2.6 $$I(y_1, y_2, \ldots, y_n) = J(Y) - \sum_i J(Y_i) + \frac{1}{2}\log\frac{\prod_{i=1}^{n}\sigma_{ii}}{\det(\Sigma)}$$

where $\sum = cov\ (Y), \sigma_{ii} = \underset{i}{diag}(\Sigma)$. If the $y_{i's}$ are uncorrelated, then (2.6) reduces to:

2.7 $$I(y_1, y_2, \ldots, y_n) = J(Y) - \sum_i J(y_i).$$

Maximizing the negentropy of Y requires knowledge of the density f(.). Approximation based on the cumulants of Y are available (Jones and Sibson (1987), Comon (1994)).

Definition 4. Let $Z = \frac{Y-E(Y)}{\sigma_y}$ where $E(Y) = \int Yf(y)dy$ and $\sigma_y^2 = E(Y-\mu_y)^2$. Then, the skewness of Y, sk(y) and Kurtosis of y or kurt(y) are:

$$sk(y) = E(Z^3)$$

$$Kurt(y) = E(Z^3) - 3$$

Jones and Sibson (1987) showed that:

$$2.8 \qquad J(y) \simeq \frac{1}{12}[sk(y)]^2 - \frac{1}{48}[kurt(y)]^2.$$

We replace *sk(y)* and *kurt(y)* by their sample estimates. Generalizing (2.7) by defining contrast function G(.) we obtain:

$$2.9 \qquad J(y) \simeq \{E[G(w_i^T x) - E(G(v))]\}^2.$$

Where v is a standardized Gaussian random variable. Some contrast function suggested include:

$$3.0 \qquad G_1(u) = \frac{1}{\alpha_1}\log\cos h(\alpha_1 u), \qquad g_1(u) = G_1^1(u) = \tan h\,(\alpha_1 u)$$

$$G_2(u) = \frac{1}{\alpha_2}\exp(-\alpha_2 u^2/2), \qquad g_2(u) = G_2^1(u) = u\,\exp(-\alpha_2 u^2/2)$$

$$G_3(u) = \frac{1}{4}u^4 \quad , \qquad g_3(u) = u^3$$

Where $\alpha_1 \geq \alpha_2 \simeq 1$ are constants.

3.0 Methods in Model Formulation

We assume that there is a commodity C which is not traded in the market. In this economic model, there is an "owner" of the commodity C and a potential buyer both of whom put a value P to it. The "owner" expresses his price for the commodity in terms of his willingness to accept compensation (WTA) given by:

3.1 $WTA = \alpha_{11} P + \alpha_{12} F.$

Where F is an adjustment factor independent of P. Once the "Owner" expresses his WTA, the buyer puts forth his counter proposal in terms of his willingness-to-pay (WTP):

3.2 $WTP = \alpha_{21} P + \alpha_{22} F.$

In principle, WTA≥WTP and negotiations ensue by the "owner" lowering his WTA and the buyer raising his WTP. A transaction occurs when WTA = WTP. The components P and F are random quantities depending on the buyer-seller pair for the same commodity.

Let $X = \begin{pmatrix} WTA \\ WTP \end{pmatrix}$, $A = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}$, $S = \begin{pmatrix} P \\ F \end{pmatrix}$, then (3.1) and (3.2) can be written as:

3.3 X=AS

where $corr(s) = I$, $corr(x) = \rho = \begin{pmatrix} 1 & \pi \\ \pi & 1 \end{pmatrix}$. The only quantity observed in this model is the vector X while A and S are unknown. When only the independence of the components is assumed, then the ICA approach in Section 2 can be applied to recover S by estimating $W=A^{-1}$.

3.4 $S=A^{-1}x=wx$

Factor Model. However, if we relax our restrictions by allowing a multivariate normal model for the vector S viz.
$S \underset{\sim}{d} M \cup N\ (\mu_s, \psi)$ where $\psi = \begin{pmatrix} \sigma_P^2 & 0 \\ 0 & \sigma_F^2 \end{pmatrix}$, then (3.3) can be written in the usual orthogonal factor model as:

3.5 $X-\mu_x=AS.$

Taking the correlation matrices of both sides of (3.5), we obtain

3.6 $corr(X-\mu)=A\ corr(s)\ A^T$

$$\rho= AA^T$$

The correlation matrix ρ is positive-definite and can be decomposed as:

$$\rho = PDP^T$$

$$= P\, D^{\frac{1}{2}}\, D^{\frac{1}{2}}\, P^T$$

Where P is an orthogonal matrix and D is a diagonal matrix whose diagonal elements are the eigenvalues of P. Let A=PD^□(1/2) or more explicitly:

$$A = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \sqrt{1+r} & 0 \\ 0 & \sqrt{1-r} \end{pmatrix} \text{ where } \lambda_1 = 1 + r, \lambda_2 = 1 - r$$

3.7 $$A = \begin{pmatrix} \frac{\sqrt{1+r}}{\sqrt{2}} & \frac{\sqrt{1-r}}{\sqrt{2}} \\ \frac{\sqrt{1+r}}{\sqrt{2}} & -\frac{\sqrt{1-r}}{\sqrt{2}} \end{pmatrix}.$$

The inverse is:

3.8 $$W = A^{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{1+r}} & \frac{1}{\sqrt{1+r}} \\ \frac{1}{\sqrt{1-r}} & -\frac{1}{\sqrt{1-r}} \end{pmatrix}.$$

The hidden signals can be recovered as:

3.9 $$P = \frac{1}{\sqrt{2(1+r)}} WTA + \frac{1}{\sqrt{2(1+r)}} WTP$$

$$F = \frac{1}{\sqrt{2(1-r)}} WTA + \frac{1}{\sqrt{2(1-r)}} WTP$$

Equation (3.9) can be re-expressed in terms of the eigenvalues of ρ:

4.0 $$P = \frac{1}{\sqrt{2\lambda_1}} (WTA + WTP)$$

$$F = \frac{1}{\sqrt{2\lambda_2}} (WTA - WTP)$$

In order to ensure that both P and F lie in the range 0<WTP≤WTA, Equation (4) can be normalized to yield:

4.1 $$P = \frac{1}{2} (WTA + WTP)$$

$$F = \frac{1}{2} (WTA - WTP)$$

The coefficient found in both equations is ½ regardless of the correlation between WTA and WTP. If the covariance matrix ∑ of WTA and WTP is used instead of the correlation matrix, the coefficients will change according to the values of e1 = (e11,e12) and e2 = (e21,e22) which are the eigenvectors corresponding to λ1 and λ2 of ∑ as follows:

4.2 $$P = \frac{1}{\sqrt{\lambda_1}} \left(e_{11}WTA + e_{12}WTP\right)$$

$$F = \frac{1}{\sqrt{\lambda_2}} \left(e_{21}\, WTA - e_{22}WTP\right)$$

Model Explanation. The latent value P of the commodity C and the adjustment factor F (in this latent value) figure in the expressed WTA and WTP of the seller and the buyer respectively. As the seller sets his price (WTA) he first determines the value of the commodity to him (say, P) and quickly assesses an adjustment factor (F) should the buyer haggle. The seller then combines P and F to form his WTA. On the other hand, the buyer recognizes this pricing pattern and forms his own combination of P and F to express his WTP hoping to match the seller's WTA. In effect, the negotiation reduces to a search of the coefficients $(\alpha_{11}, \alpha_{12}) \in R^2$ for the seller and $(\alpha_{21}, \alpha_{22}) \in R^2$ for the buyer. These coefficients are, in fact, found in the subspace spanned by the orthogonal eigenvectors (e_1,e_2) of the correlation matrix ρ namely

$$e_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \text{ and } e_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}.$$

For instance, suppose that at time t=1, the seller sets $WTA^{(1)}$ and the buyer gives $WTP^{(1)}$, $WTP^{(1)}<WTA^{(1)}$. At time t =2, the seller adjusts his *WTA* to $WTA^{(2)}$ less than $WTA^{(1)}$ to which the buyer adjusts his *WTP* to $WTP^{(2)}$ greater than $WTP^{(1)}$. These imply that $WTA^{(1)}$-$WTP^{(1)}$>$WTA^{(2)}$-$WTP^{(2)}$, and the seller gets an idea about the assumed P of the buyer and vice-versa, the buyer gets an idea about the assumed P of the seller. The process generates a monotone decreasing sequence of price differences:

$$WTA^{(1)} - WTP^{(1)} > WTA^{(2)} - WTP^{(2)} > \cdots > WTA^{(k)} - WTP^{(k)}$$
converging to zero when $WTA^{(n)} - WTP^{(n)} = 0$.

4.0 Numerical Simulations

We performed simple numerical simulation by generating 100 pairs of WTA and WTP with correlation r =0.709 from a bi-variate normal with mean equal to 150 for WTA and 80 for WTP and standard deviation of 5 for WTA and 1.3 for WTP.

Table 1 shows the average latent values and adjustment factors obtained using the correlation matrix as inputs:

Table 1: Average Latent Value and Adjustment Factor with Correlation Input

Variable	N	Mean	Median	TrMean	StDev	SE Mean
WTA	100	149.80	149.56	149.74	4.87	0.49
WTP	100	79.857	9.936	79.873	1.269	0.127
Value	100	114.83	114.79	114.80	2.92	0.29
Factor	100	34.970	34.895	34.952	2.035	0.203

Eigenanalysis of the Correlation Matrix

Eigenvalue	1.7088	0.2912
Proportion	0.854	0.146
Cumulative	0.854	1.000

Variable	PC1	PC2
WTA	0.707	0.707
WTP	0.707	-0.707

Table 1 shows result of the descriptive analysis to the average latent value and adjustment factor with correlation input. After analyzing the Correlation Matrix, it has been determined that the acceptance level of the variance should be at least 80%. The cumulative value obtained is 100%, which indicates that the level of variance is accepted. Therefore, the two variable will be use to analyze the estimation of the latent value. The size of the eigenvalue also determine the number of principal components. Retain the principal components with the largest eigenvalue that is greater than 1. The first principal component (PC1) accounts for 85.4% of the total variance. The variables that show the highest correlation with the first principal component are WTA and WTP, with a correlation coefficient of 0.707 each. The first principal component is positively correlated with the WPA and WTP variables.

The graph of these values is shown below:

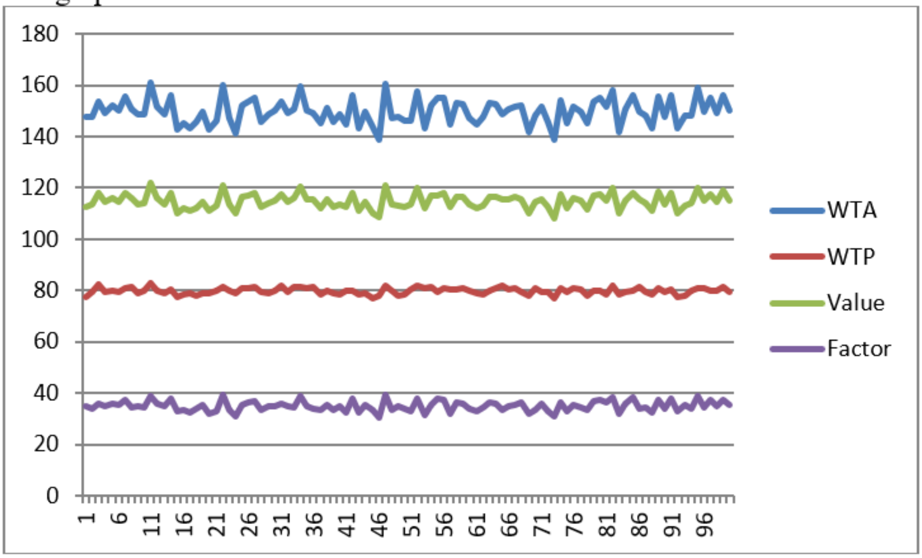

Figure 1: WTA,WTP, Latent Value and Adjustment Factor Obtained from Correlation Eigen-analysis

Figure 1 presents the graph of the numerical simulation result of the WTA, WTP, Latent Value, and the Adjustment Factor Obtained from the Correlation Eigenanalysis. The estimated true value is 115 with an adjustment factor of around 35 given WTA of 150 and WTP of 80.

Table 2 shows the average latent values and adjustment factor with the covariance matrix as input:

Table 2: Average Latent Value and Adjustment Factor with Covariance Input

Variable	N	Mean	Median	TrMean	StDev	SE Mean
WTA	100	149.80	149.56	149.74	4.87	0.49
WTP	100	79.857	79.936	79.873	1.269	0.127
Value(co	100	138.56	138.40	138.51	4.23	0.42
Factor(c	100	112.89	112.55	112.85	3.94	0.39

Eigenanalysis of the Covariance Matrix

Eigenvalue	24.548	0.774
Proportion	0.969	0.031
Cumulative	0.969	1.000

Variable	PC1	PC2
WTA	0.982	-0.188
WTP	0.188	0.982

Table 2 shows result of the descriptive analysis to the average latent value and adjustment factor with covariance input. Upon analyzing the Correlation Matrix, the two components explain 100% of the total variation in the data. Therefore, the two variable will be use to analyze the estimation of the latent value. The size of the eigenvalue also determine the number of principal components. Retain the principal components with the largest eigenvalue that is greater than 1. The first principal component (PC1) accounts for 96.9% of the total variance. The variables that show the highest correlation with the first principal component are WTA and WTP, with a correlation coefficient of 0.982 each. The first principal component is positively correlated with the WPA and WTP variables.

Figure 2 shows the graph of these values:

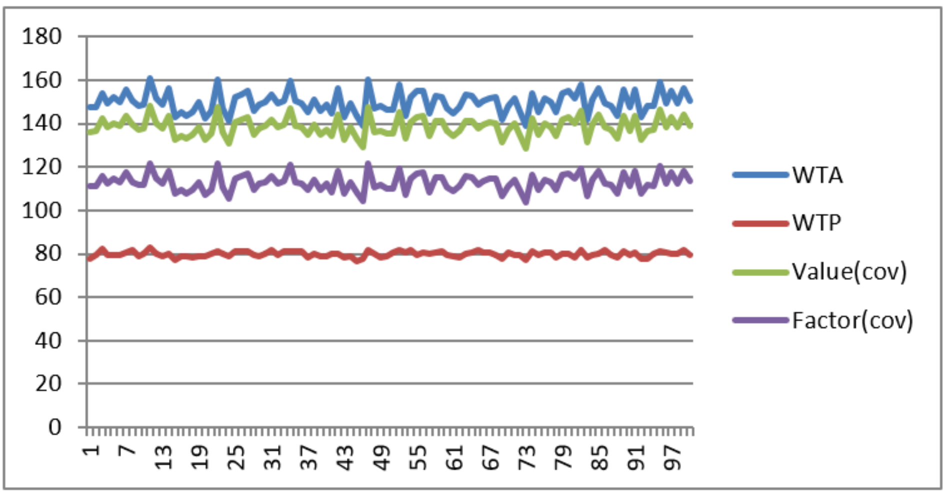

Figure 2: WTA,WTP, Latent Value and Adjustment Factor Obtained from Covariance Eigen-analysis

Figure 2 presents the graph of the numerical simulation result of the WTA, WTP, Latent Value, and the Adjustment Factor Obtained from the Covariance Eigenanalysis. The estimated true value is 138 with an adjustment factor of around 112.89 given WTA of 150 and WTP of 80. The concepts of "willing to pay" (WTP) and "willing to accept" (WTA) are often used to assess individuals' preferences and choices regarding gains and losses. When an estimated values are closer to WTA compensation, it indicate that the estimates better capture individuals' preferences and valuations in terms of what they are willing to accept as compensation.

Conclusion

In economics and decision theory, the concepts of "willing to pay" (WTP) and "willing to accept" (WTA) are often used to value individuals' preferences and select regarding gains and losses. Valuing goods and services not normally traded in the market. The ICA platform as a means to estimate the latent price P in contingent valuation. The two unknown independent signals are estimated by Independent Component Analysis (ICA) and by Factor Analysis methods. Knowledge of the latent value and adjustment factor aids in the negotiation process in contingent valuation in environmental economics.

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