USING THE DYNAMICS OF THE LOGISTIC MAP AS TOOL FOR RAPID ECOSYSTEM ASSESSMENT

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ABSTRACT

This paper uses the logistic map as a primary tool for Rapid Ecosystem Assessment (REA). The behavior of the iterates of the logistic map depends on the growth parameter θ which conveys information about how well the ecosystem supports the growth of the organism. The ratio $\tau = \frac{\theta}{4}$, expressed in percentage, indicates the proportion of the environment which is still in good condition. An estimator for θ based on available data is given as 4y, where y is the largest observed value. The properties of the estimator are derived and discussed. Data on the fisheries volume of production by Region and by Province were obtained from the Fisheries Statistics of the Philippines from 2001 to 2015 on which the growth parameters were estimated. Results indicate that in the Caraga region, 96% of the marine environment remains good and capable of sustaining fisheries. On the other hand, Region IV-A has only 70% of its fishing grounds, staying in excellent condition while the remaining 30% of the fishing ground is no longer able to sustain fisheries production. The effects of industrialization and overfishing in Region IV-A (Calabarzon Special Economic Zone) were identified as possible causes of the gradual decline of the excellent fishing grounds in this region. More rigorous Marine Environment Assessment to pinpoint the 30% degraded area and the establishment of Marine Protected Areas to resuscitate these fishing grounds are suggested as interventions.

Keywords: *logistic map, chaos, stable fixed points, time series, marine protected area*

1.0 Introduction

Environmental assessment or ecosystem assessment is often required by authorities to ensure that resource development projects e.g. land use, do not impact negatively on biodiversity. Such assessment procedures involve detailed protocols for determining the status of biological and non-biological environment parameters. Unfortunately, such environmental assessment protocols are time-consuming and costly in many cases (White, 1998). Alcala (2001) produced a comprehensive assessment of the status of Marine Protected Areas (MPA) in Central Visayas in a two-year funded project by the Department of Foreign Affairs (DFA). However, when it is desired to obtain quick preliminary information about an ecosystem, rapid assessment tools need to be developed. This paper deals with the logistic map as a primary tool for Rapid Ecosystem Assessment (REA).

May (1976) first investigated the logistic map:

$$x_{n+1} = \theta x_n(1 - x_n) \quad , \; 0 < \theta \le 4, \; 0 < x_n \le 1 \tag{1}$$

where x_n represents the number of organisms in an environment divided by the carrying capacity M of the environment and θ is a growth parameter that conveys information about how well the ecosystem supports the growth of the organism. The logistic dynamics of population growth were observed by May (1976) under laboratory conditions for fruit-flies. He found that the parameter θ controls the behavior of the population dynamics of the flies. For instance, when the environment was so designed as to suppress the growth of the flies, e.g. controlling the food source of the organisms i.e. the value of θ ranges from 0 to 1, the population eventually becomes extinct.

Thus, in practice, the value of θ is a useful indicator of the environmental conditions supporting the growth of biological organisms. Hayes (1993), in a US Defense Department funded project, provided maximum-likelihood estimator of θ based on the model:

$$x_{n+1} = \theta x_n(1 - x_n) + \varepsilon_{n+1} \tag{2}$$

where the $\varepsilon' s$ are random noise from a known symmetric distribution F(.).

His estimator is given by:

$$\widehat{\theta} = \sum w_i \,\widehat{\theta_i} \quad , \quad \sum w_i = 1, \qquad \widehat{\theta_i} = \frac{x_{i+1}}{x_i\,(1-x_i)} \tag{3}$$

and was shown to be an asymptotically efficient estimator of θ. In this paper, we propose a non-parametric counterpart of (3) and demonstrate its statistical properties as well. The main advantage of the proposed estimator in this paper over that of Hayes (1993) is the ease with which one can apply the estimator to actual time-series data.

2.0 Parameter Estimation Model

The logistic map (1) illustrates varying population behavior as the growth parameter θ is varied. When $0< \theta \leq 1$, the population $\{x_n\}$ tends to zero eventually, $X_n \rightarrow 0 \; as \; n \rightarrow \infty$; when $1< \theta \leq 3$, the population $\{x_n\}$ settles to a single periodic points $\frac{\theta-1}{\theta}$ i.e. $X_n \rightarrow \frac{\theta-1}{\theta}$ as $n \rightarrow \infty$. Thus, when $\theta = 2$ the population $\{x_n\}$ eventually occupies $\frac{\theta-1}{\theta} = \frac{1}{2}$ or 50% of the environment's maximum carrying capacity. For $\theta > 3$, the population settles to period 2 cycles, period 4 cycles, period 8 cycles..., period 2^n cycles until it becomes chaotic for $=$ 4. Chaotic systems are characterized by unpredictable periods of population "busts" and "booms" and, further, describe the growth of biological organisms in the wild with a pristine environment.

When $\theta = 4$, the logistic map:

$$X_{n+1} = 4X_n \; (1\text{-}X_n) \qquad (4)$$

is chaotic and the realizations $\{x_n\}$ behave like random quantities. Applying the Frobenius –Perron operator (Devaney, 1997), these pseudo-random numbers obey a beta distribution with $\propto= \frac{1}{2}, \beta = {}^{1}\!/_{2}$ given by:

$$\rho(x)=\frac{1}{\pi\sqrt{x(1-x)}} \quad , \quad 0< x < 1 \qquad (5)$$

The set of all attractors of (4) is the interval $[0,1]$ and, in general, the set of attractors of (1) is the interval $\left[0, \frac{\theta}{4}\right]$. Using this information, we obtain the first non-parametric estimate θ :

$$\hat{\theta} = 4 \; \max \; \{x_n\} \qquad (6)$$

Estimator (6) is intuitively appealing. For instance, under a chaotic regime, the chaotic attractors lie on the interval $[0,1]$, hence, max $\{x_n\}$ =1 and $\widehat{\theta_\iota}$ =4 . 1 =4.

When the maximum number of the organism is 50% or ½ of the carrying capacity, then $\hat{\theta}$= (4)(½) = 2 as earlier shown.

However, given a time series $\{x_t\}$ of observations, the serial sequence x_t *and* x_{t+1} need not be related by the logistic map (1). These observations may be thought of as being "embedded" in the space of observations generated by the logistic map. The problem is now to recover the embedding space, that is, find the appropriate logistic map that the observations may have come from. More succinctly, find θ in Equation (1) given the time series $\{x_t\}$.

In order to accomplish this, we need to treat the realizations of Equation (1) as random quantities that obey a probability distribution. In particular, for values of θ not equal to 4, we consider the limiting distribution of these realizations. We begin by defining terms:

Definition 1. Let f be a continuous one-dimensional map such that:

$$x_{t+1} = f(x_t),$$

x* is a **fixed point** of f iff $x^* = f(x^*)$.

When θ is between 0 and 1 in the logistic map, the fixed point is x* = 0; when it is between 1 and 3, it has two fixed points x* = 0 and $x^* = \frac{\theta-1}{\theta}$. The fixed point x* = 0 is **unstable** and repels points away from it, while the other fixed point is **a stable fixed point** and attracts points to it. These are periodic points of period 1.

Definition 2. Let f be a continuous one-dimensional map such that:

$$x_{t+1} = f(x_t),$$

x* is a **fixed point** of f of period n iff $x^* = f^n(x^*)$ and n is the smallest positive integer.

When θ is between 3 and 3.4494, the logistic maps has a period 2 orbit, namely:

$$x^* = \frac{(1+\theta) \pm \sqrt{\theta^2 - 2\theta - 3}}{2\theta} \tag{7}$$

Starting from any initial value x_0 , the successive iterates eventually toggles between these two values. For $\theta > 3.4494$, period 4, period 8, period 16,..., period 2^n fixed points are observed.

For large n, and when $1 < \theta \leq 3$, the limiting distribution of the iterates is degenerate at $\frac{\theta-1}{\theta}$.

$$P(x = \frac{\theta-1}{\theta}) = 1 \qquad (8).$$

When $3 < \theta \leq 3.4494$, the limiting distribution of the iterates is a Bernoulli distribution:

$$P\left(x = \frac{(1+\ \theta) + \sqrt{\theta^2\ -\ 2\theta - 3}}{2\theta} \right) = \ p$$

$$P\left(x = \frac{(1+\ \theta) - \sqrt{\theta^2\ -\ 2\theta - 3}}{2\theta} \right) = \ 1 - p \qquad (9)$$

For larger values of θ, the limiting distribution of the iterates is a multinomial distribution. It can be shown that for the logistic map, when the number of periodic points is infinite via period doubling bifurcation, the limiting distribution is equal to its ergodic density which is the beta distribution previously given.

Consider the specific case in which there is a period 2 orbit and the iterates toggle between the smaller and the larger periodic points. Denoting the smaller periodic point by x and the larger by y, a time series realization of this situation may consists of the following $\{x, x, y, x, y, y, , y, x, x, y, y, y, ...\}$ towards the tail end of the series, in contrast to the theoretical behaviour using the logistic map which is either $\{x, y. x, y, x, y, x, y, ...\}$ or $\{y, x, y, x, y, x, y, x, ...\}$. Using Equation (6) as an estimator of θ, we have

$$\hat{\theta} = 4\ \max\ \{x_n\}\ = 4y$$

However, $y \neq max\{x_n\} = \frac{\theta}{4}$. Thus,

Lemma 1. The estimator $\hat{\theta} = 4\ \max\ \{x_n\}$ is a biased estimator of θ.
Proof. As shown above.

We note, however, that $y = max\left\{x_{p_j}\right\}\ over\ j$ where x_{p_j} is a periodic point of period n of the logistic map. Let:

$$\gamma = \ |\frac{\theta}{4} - y|$$

Note that γ is the distance between the largest value of $\{x_t\}$ and the largest fixed point.

Lemma 2. The quantity:

$$\gamma = \left| \frac{\theta}{4} - y \right| \leq \frac{1}{2^{n-1}}\left(\frac{\theta}{4}\right)$$

where n is the number of stable fixed points of the logistic map.

Proof. The degenerate case when the fixed point is zero is obviously satisfied i.e. y = 0 and n =1. Similarly, when the single stable fixed point is $\frac{\theta-1}{\theta}$, then

$$\gamma = \left| \frac{\theta}{4} - \frac{\theta - 1}{\theta} \right| = \left| \frac{\theta^2 - 4\theta + 4}{4\theta} \right| \leq \left(\frac{\theta}{4}\right)$$

When there are two stable fixed points, then one fixed point is to the left of $\frac{\theta}{8}$ and the other is to the right of it. Hence,

$$\gamma = \left| \frac{\theta}{4} - y \right| \leq \frac{1}{2}\left(\frac{\theta}{4}\right)$$

Continuing in this fashion, we find that if there are n stable fixed points, then half of them will be to the left of $\frac{1}{2^{n-1}}\left(\frac{\theta}{4}\right)$ and half to the right.■

Taking the largest possible value of $\theta = 4$, we find an estimate of γ that is independent of θ. The estimator (6) can now be modified with less bias, namely:

$$\hat{\theta} = 4\,(y + \gamma) \qquad (10)$$

Moreover, the bias tends to zero as the number of stable fixed points increases to infinity.

Corollary. The quantity:

$$\gamma(n) = \left| \frac{\theta}{4} - y \right| \to 0 \; as \; n \to \infty$$

where n is the number of stable fixed points of the logistic map.

3.0 Estimation of the Maximum Carrying Capacity M and Sample Limit Distributions

In estimator (6), the input information is $\{x_n\}$ which are scaled values of the actual number of organisms divided by the maximum carrying capacity M. The estimators are sensitive to changes in the values of M, hence, a reliable estimate M is needed.

Assuming natural processes, we expect that:

$$P(-2.576 < \frac{m-E\,(m)}{\sigma(M)} < 2.576\,) = 99.9\,\% \qquad (11)$$

Hence, the maximum carrying capacity is $\widehat{M} = E\,(m) + \,2.576\,\sigma(m)$ with 99.9 % probability.

The succeeding figures show the limiting distribution of the iterates of the logistic map for different values of θ and increasing number of stable fixed points. Note that all the biases are less than $\gamma = 0.001$ and decreases with increasing number of stable fixed points.

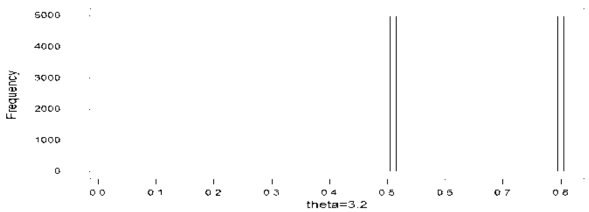

Figure 1: Limiting Distribution with Two Stable Fixed Points
Largest Fixed Point:0.79946, Maximum Possible Value: 0.8000, Bias: 0.00054

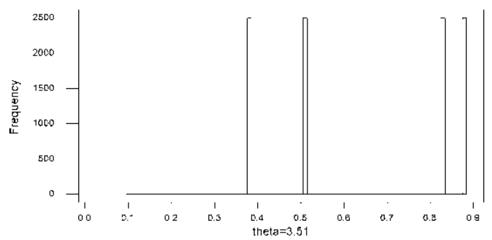

Figure 2: Limiting Distribution with Four Stable Fixed Points
Largest Fixed Point: 0.87741, Maximum Possible Value: 0.8775, Bias: 0.00009

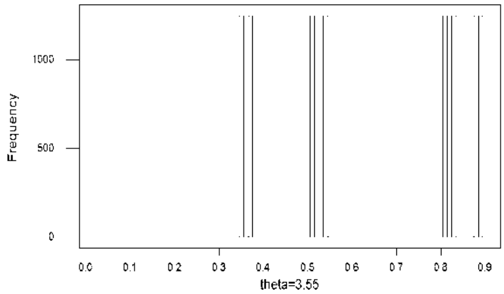

Figure 3: Limiting Distribution with Eight Stable Fixed Points
Largest Fixed Point: 0.88746, Maximum Possible Value : 0.88750, Bias:0.00004

4.0 Rapid Assessment of the Fisheries Ecosystem in the Philippines

Data on the fisheries volume of production by Region and by Province were obtained from the Fisheries Statistics of the Philippines (Philippine Statistics Authority, Vol.12-24, 2016) from 2001 to 2015.

Table 1 shows the estimated growth parameter θ for each of the regions of the country. Tabular values show that the best fishing grounds are, in order of "wellness", Caraga, Region VI (Western Visayas) and Region IX (Zamboanga Peninsula). Of the top three fishing grounds, Caraga registered a growth parameter of 3.8282 which indicates that around 96% of the marine fishing environment of this region is in an excellent condition while around 4% has been degraded. Western Visayas, on the other hand, boasts of 93% good fishing ground with only 7% of the marine environment ill-suited for sustaining fisheries in the region. Zamboanga Peninsula, the country's main area for sardines, has 91% intact and good fishing ground while 9% of the fishing area is potentially in peril. We note that two of the three top fishing grounds in the country are in Mindanao while the third is in the Visayas region.

In contrast, the three least fit fishing grounds are Region IV-A (Calabarzon), Region IV-B (Mimaropa) and Region 8. In particular , the fishing grounds in Region IV-A or the Calabarzon area are in "critical condition" with estimated growth parameter of 2.7700 which means that the area can only support 69% of the carrying capacity of the marine environment there, unless something is done to improve the conditions in this area. A little less than a third of the fishing ground (30%) in the region is incapable of sustaining fish production in the region. Conditions in the contiguous region, Region IV-B or the MIMAROPA are significantly better than in Region IV-A posting a growth parameter of 3.1088 which means that 78% of the fishing ground in this region remains good but 22% of the marine environment in this region is no longer productive. Region 8 or the Eastern Visayas region follows very closely registering a growth parameter of 3.1296 indicating that 78% of the fishing area is still in good condition similar to that of Region IV-B. . It is noted that two of the least fit marine environments are located in Luzon while only one is found in the Visayas area. The poor showing of the Eastern Visayas region may be attributed to the damage brought about by Typhoon Yolanda in 2013 which severely damaged the marine ecosystem of the surrounding waters of the Samar and Leyte provinces. Regions IV-A and IV-B are special economic zones with massive industrialization, which consequently adversely impacted on the surrounding fishing grounds. Overfishing and marine exploitation of the fishing grounds in these three areas cannot be ruled out as possible explanations as well.

Table 1: Growth Parameter Theta Based on the Maximum Estimate

Variable	Mean	St. Dev.	M	Theta (Ɵ)	Percent of Fishing Ground Suitable for Fisheries	Rank
CAR	3643	389	4645	3.6416	91.04%	4
I	126696	35009	216879	3.2128	80.32%	12
II	54535	9098	77971	3.328	83.20%	9
III	238593	41064	344374	3.2296	80.74%	11
NCR	112903	32291	196085	3.428	85.70%	8
4A	351906	96371	600157.7	2.77	69.25%	17
4B	597448	137786	952385	3.1088	77.72%	16
V	241243	54663	382055	3.1444	78.61%	14
VI	402455	31115	482607	3.7372	93.43%	2
VII	215524	19910	266812	3.6076	90.19%	5
VIII	161627	42032	269901	3.1296	78.24%	15
IX	577078	97383	827937	3.6585	91.46%	3
X	136753	26535	205107	3.1748	79.37%	13
XI	60877	7454	80079	3.5012	87.53%	6

XII	271814	42272	380707	3.4536	86.34%	7
ARMM	753666	139705	1113546	3.2664	81.66%	10
Caraga	95934	7847	116147.9	3.8282	95.71%	1

Figure 4 shows the time series plot of the fishery production for the top three best marine environments in the country. The plot suggests that the Caraga region's fishing grounds are least disturbed judging from the low fishery outputs, hence, explaining the excellent growth parameter earlier computed. On the other hand, the second and third best fishing grounds have far greater fishery production indicating significantly greater disturbance to the marine ecosystems of these two areas, namely, Region VI and Region IX. From these observations, one may deduce that the growth parameter θ may also be used as an indicator of the degree of disturbance experienced by the ecosystem: the greater the value of the growth parameter, the lesser is the disturbance of the environment and conversely.

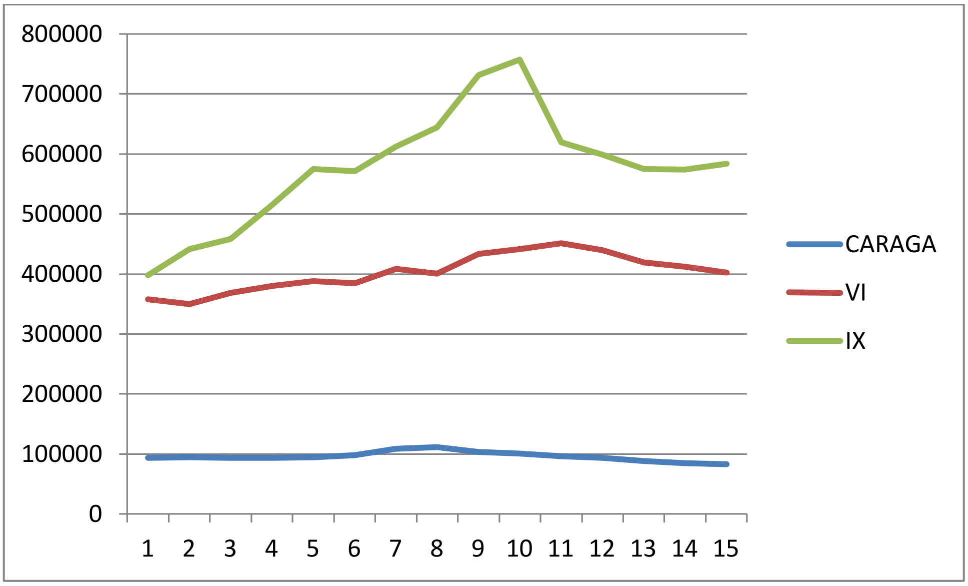

Figure 4: Fishery Production of the Top Three Fishing Grounds

Figure 5 shows the time series plot of the fishery production of the three least fit fishing grounds in the country. Region VIII has the least fishery production among the 3 regions wtith the least fit fishing grounds. This means that among the three regions, the fishing grounds of Region 8 had been least exploited and hence, least disturbed as well Consequently, one would expect that the growth parameter in this region would be the highest of the three regions. Indeed, the computed growth parameter for Region 8 posted the largest value of 3.1296. The cases of Regions IV-A and IV-B tell a slightly different story. While the time series plot indicates that Region IV-B had higher fishery production than Region IV-A and so, one would expect the growth parameter in Region IV-B to be lower than that of Region IV-A. However, this is not the case since the growth parameter for Region IV-A is 2.7700 as compared to Region IV-B which registered a higher value of 3.1088. This phenomenon can be explained when we consider the areas of the fishing grounds of the two regions. Region IV-A has a significantly lesser fishing area than Region IV-B. Thus, for the same number of fishermen, Region IV-B would expectedly yield greater fishery production. That is, a more refined characterization of the wellness of the marine environment would have to take into account both the growth parameter θ and the area of the fishing ground:

$$fishery\ production\ = f(growth\ parameter, area\ of\ fishing\ ground) \quad (11)$$

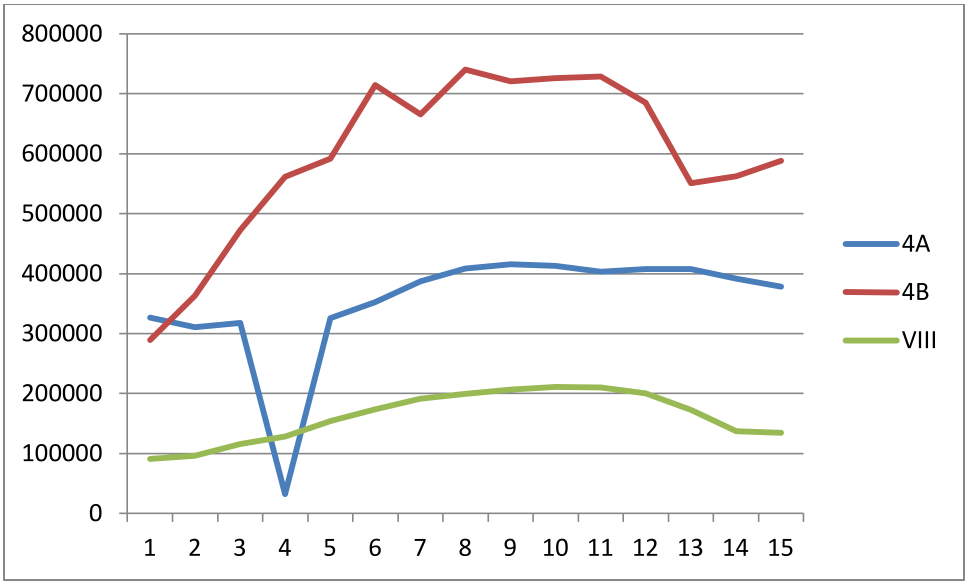

Figure 5: Time Series of Fisheries Production of the Lowest Three Regions

5.0 Conclusion

The study showed the efficacy of using the growth parameter of the logistic mapping in providing a rapid assessment of an ecosystem. The technique can be used as a preliminary rapid assessment of a biological ecosystem which can then be followed by a more detailed environment assessment protocol. The technique requires information on the number of biological organisms recorded as a time series. The study also hints on the possibility of expressing the biological production as a function of the growth parameter and the area of the biological ecosystem under study.

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