METHODS FOR EXTRACTING TRUE ABILITY AND CONFUSION IN STUDENTS' TEST SCORES

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ABSTRACT

This paper explores a new method for extracting true ability and confusion in students' test scores. Employing an Eigen analysis, the test scores of the students in Test1 and Test 2 respectively, are processed using the covariance matrix in order to determine the true ability and confusion of the students. Considering that the students get high scores; when variations exist by getting a lower score in their Test 1 than in Test 2, the result appears to have a high true ability but with little confusion. However, if the latter test is lower than the first test, the outcome indicates that the students obtained high true ability as well as high confusion. Meanwhile, for the students with low scores; if their first test is higher than their second test, the students can be interpreted with little ability and very confused; and if their Test 1 is lower than their Test 2, the students' test scores are just a result of their guess works. On the other hand, if the students' scores in the first test are equal to their last test, the test scores reflect the students' true ability with no confusion. Hence, this concept is a realistic method of measuring the true ability and confusion of the students.

Keywords: true ability, confusion, Eigen analysis, test scores

Introduction

Classical test theory espouses the model:

1.1 Score = True ability + random error

where the random errors cancel over repeated testing (Thorndike, 1957). This model justifies the use of the mean score as a measure of the student's true ability. However, the model also assumes that a person's true ability is a fixed parameter independent of the kind and nature of the testing done. In this paper, we explore the Bayesian point of view where, instead of treating the "true ability (TA)" as a parameter, we consider it as a random variable independent of the random error (Graybill, 1976; Johnson and Wichern, 2000).

Several authors (Rubio, 2014; Dales, 2007) have criticized the classical test model on the grounds of “stereotyping” students into various categories viz. fast and slow learners; bright and dull. Instead of serving as a motivator, tests are feared by many students and are viewed as necessary “evil” in the student’s education process. The psychological impact of low grades on students has been thoroughly investigated by researchers (Rubio, 2014; Dales, 2007). In these studies, students receiving low grades because of poor test performance were found to possess low self-esteem and developed poor attitudes toward learning. On the other hand, low self-esteem and lack of interest in academic studies were found to have an adverse effect on student’s performance on various stages in their academic studies (Sumbalan, 2008; Rubio, 2014). In other words, the classical perspective on testing and evaluation induces a cycle whose end-result impacts negatively on a child’s educational experience.

The studies reviewed showed that treating a student’s ability as a fixed parameter has negative consequences. On the other hand, no serious research has been done to serve as an alternative to classical test model. In this paper, we explore the possibility of a random true ability score that depends on the nature and quality of the test given. We contend that viewpoint, namely, a Bayesian perspective is both fair and realistic and breeds minimal negative consequences on a child’s self-concept or self-esteem. It is consistent with a child’s self-concept or self-esteem. It is likewise consistent with the principle of Outcomes-Based Education.

Model Formulation

The researchers assume that there are scores of student tests whenever there is a test. The scores contain “True Ability” and “Confusion.” Thus, the test 1 score is modeled as:

1 $$Test\ 1 = \alpha_{11}TA + \alpha_{12}C.$$

Similarly, the retest or Test 2 can be expressed as:

2 $$Test\ 2 = \alpha_{21}TA + \alpha_{22}C.$$

The components TA and C are random quantities depending on the scores of the students.

Let $X = \begin{pmatrix} Test\ 1 \\ Test\ 2 \end{pmatrix}$, $A = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}$, $S = \begin{pmatrix} TA \\ C \end{pmatrix}$, then (1) and (2) can be written as:

3 $\quad X = AS$

where $corr(s) = I$, $corr(x) = \rho = \begin{pmatrix} 1 & \pi \\ \pi & 1 \end{pmatrix}$. The only quantity observed in this model is the vector X while A and S are unknown. When only the independence of the components is assumed, then the ICA approach can be applied to recover S by estimating $W = A^{-1}$.

4 $\quad S = A^{-1}x = wx$

Factor Model. However, if we relax our restrictions by allowing a multivariate normal model for the vector S viz. $S \underset{\sim}{d} M \cup N\ (\mu_s, \psi)$ where $\psi = \begin{pmatrix} \sigma_P^2 & 0 \\ 0 & \sigma_F^2 \end{pmatrix}$, then (2.4) can be written in the usual orthogonal factor model as:

5 $\quad X - \mu_x = AS.$

Taking the correlation matrices of both sides of (2.5), we obtain

6 $\quad corr(X - \mu) = A\ corr(s) A^T$

$$\rho = AA^T$$

The correlation matrix ρ is positive-definite and can be decomposed as:

$$\rho = PDP^T$$

$$= P\ D^{\frac{1}{2}} D^{\frac{1}{2}} P^T$$

Where P is an orthogonal matrix and D is a diagonal matrix whose diagonal elements are the eigenvalues of P. Let $A = PD^{\frac{1}{2}}$ or more explicitly:

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \sqrt{1+r} & 0 \\ 0 & \sqrt{1-r} \end{pmatrix}$$

where $\lambda_1 = 1 + r$, $\lambda_2 = 1 - r$

7 $$A = \begin{pmatrix} \frac{\sqrt{1+r}}{\sqrt{2}} & \frac{\sqrt{1-r}}{\sqrt{2}} \\ \frac{\sqrt{1+r}}{\sqrt{2}} & -\frac{\sqrt{1-r}}{\sqrt{2}} \end{pmatrix}.$$

The inverse is:

8 $$W = A^{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{1+r}} & \frac{1}{\sqrt{1+r}} \\ \frac{1}{\sqrt{1-r}} & -\frac{1}{\sqrt{1-r}} \end{pmatrix}.$$

The hidden signals can be recovered as:

9 $$TA = \frac{1}{\sqrt{2(1+r)}}\, Test1 + \frac{1}{\sqrt{2(1+r)}} Test2$$

$$C = \frac{1}{\sqrt{2(1-r)}}\, Test1 + \frac{1}{\sqrt{2(1-r)}}\, Test2$$

Equation (9) can be re-expressed in terms of the eigenvalues of ρ:

10 $$TA = \frac{1}{\sqrt{2\lambda_1}}\, (Test1 + Test2)$$

$$C = \frac{1}{\sqrt{2\lambda_2}} (Test1 - Test\ 2)$$

In order to ensure that both TA and C lie in the range 0<Test1≤Test2, Equation (4) can be normalized to yield:

11 $$TA = \frac{1}{2}\, (Test1 + Test2)$$

$$C = \frac{1}{2} (Test1 - Test2)$$

The coefficient found in both equations is ½ regardless of the correlation between Test1 and Test 2. If the covariance matrix $\sum$ of Test1 and Test 2 is used instead of the correlation matrix, the coefficients will change according to the values of $\mathbf{e_1} = (e_{11}, e_{12})$ and $\mathbf{e_2} = (e_{21}, e_{22})$ which are the eigenvectors corresponding to λ_1 and λ_2 of $\sum$ as follows:

12 $$TA = \frac{1}{\sqrt{\lambda_1}}\, (e_{11} Test1 + e_{12} Test\ 2)$$

$$C = \frac{1}{\sqrt{\lambda_2}}(e_{21}\,Test1 - e_{22}Test\,2)$$

Model Explanation

The true ability and confusion of the students are expressed in Test1 and Test 2, respectively. Considering that the students get high scores, if variations exist by getting a lower score in their Test 1 than in Test 2, the result implies high true ability but with little confusion. However, if the latter test is lower than the first test, the outcome indicates that the students have a high true ability as well as high confusion. Meanwhile, for the students with low scores, if Test 1 is higher than their Test 2, the students can be interpreted with little ability and very confused; and if their Test 1 is lower than their Test 2, the students' test scores are just a result of their guess works. On the other hand, if the students' scores in the first test are equal to their last test, the test scores reflect the students' true ability with no confusion.

Numerical Simulations

The researchers performed a simple numerical simulation by generating 10 pairs of TS^1 and TS^2 of students with above average IQ, average IQ, and poor IQ.

Table 1 shows the Test Scores values and adjustment factors obtained using the covariance matrix as inputs:

Table 1. True Ability and Confusion of Students' Tests Scores with Covariance Input (Above Average IQ)

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Test Sco	10	90.500	90.000	90.500	1.716	0.543
TA	10	92.256	92.765	92.341	1.767	0.559
Test Sco	10	93.600	94.000	93.750	2.066	0.653
C	10	10.662	10.790	10.722	0.739	0.234

Eigen analysis of the Covariance Matrix

Eigenvalue	6.1378	1.0733
Proportion	0.851	0.149
Cumulative	0.851	1.000

Variable	PC1	PC2
Test Sco	0.608	0.794
Test Sco	0.794	-0.608

The graph of these values is shown below:

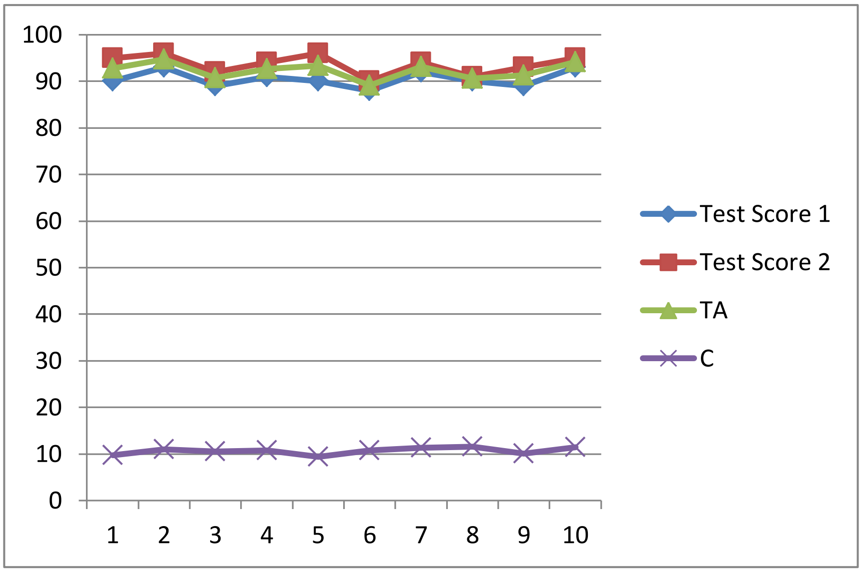

Figure 1. TA, C, Test Score 1 and Test Score 2 Obtained from Correlation Eigen-analysis (Above-average IQ)

Table 2. True Ability and Confusion of Students' Tests Scores with Covariance Input (Average IQ)

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Test Sco	10	84.200	84.500	84.000	2.821	0.892
Test Sco	10	84.200	83.500	84.250	2.658	0.841
C	10	4.055	3.976	4.069	1.199	0.379
TA	10	84.200	83.810	83.988	2.468	0.780

Eigen analysis of the Covariance Matrix

Eigenvalue	12.155	2.868
Proportion	0.809	0.191
Cumulative	0.809	1.000
Variable	PC1	PC2
Test Sco	0.740	-0.672
Test Sco	0.672	0.740

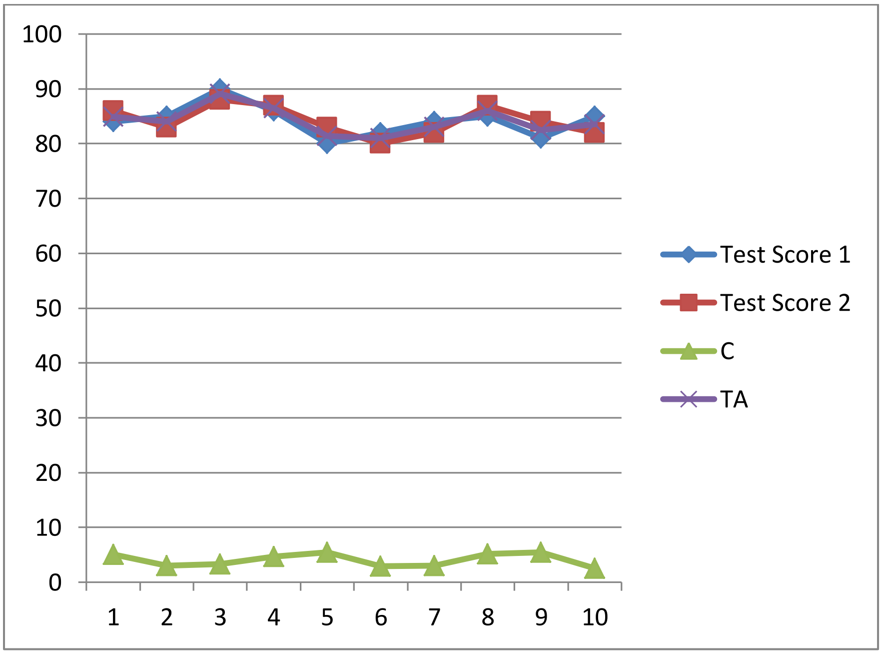

Figure 2. TA, C, Test Score 1 and Test Score 2 Obtained from Correlation Eigen-analysis (Average IQ)

Table 3. True Ability and Confusion of Students' Tests Scores with Covariance Input (Poor IQ)

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Test Sco	10	52.90	52.50	53.13	12.78	4.04
Test Sco	10	50.10	50.50	50.00	14.11	4.46
TA	10	51.43	51.45	51.66	13.41	4.24
C	10	3.988	3.951	4.072	1.309	0.414

Eigen analysis of the Covariance Matrix

Eigenvalue	358.11	7.24
Proportion	0.980	0.020
Cumulative	0.980	1.000

Variable	PC1	PC2
Test Sco	0.708	-0.707
Test Sco	0.707	0.708

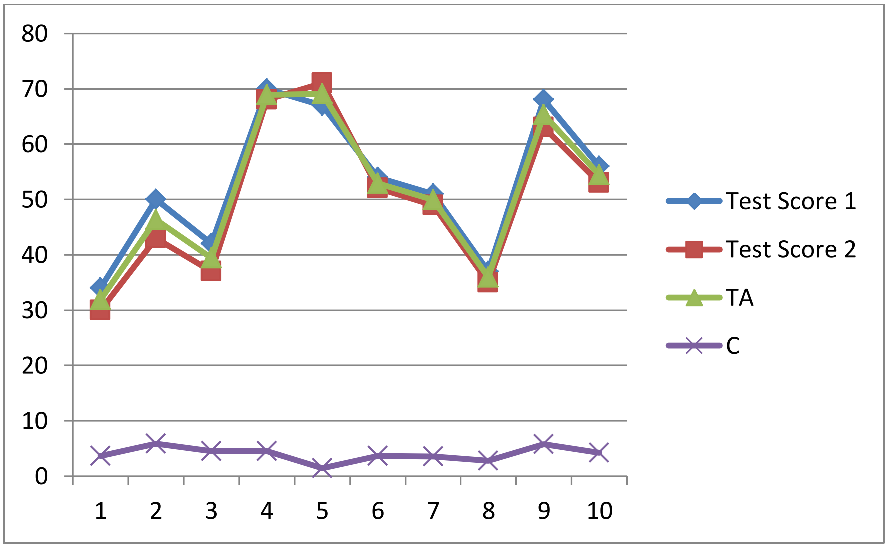

Figure 3. TA, C, Test Score 1 and Test Score 2 Obtained from Correlation Eigen-analysis (Poor IQ)

Conclusion

In this concept, the true ability and confusion of the students are determined depending on their test scores. In effect, the students who have limited ability and with high confusion rate are easily recognized. Hence, it would be easier for the teacher to recognize these learners and create remedial classes to optimize their ability while reducing their confusion. Thus, this method for extracting students' test scores employing an Eigen analysis is a realistic scheme for measuring the true ability and confusion of the students.

As this method appeared to be consistent with the principle of Outcomes-Based Education, it is, therefore, rational to utilize this method in evaluating and/or grading the academic performance of the students both in basic and higher education in the country. In addition, this method would also contribute significantly in eradicating drop-out students due to the failing of grades.

References

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