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THEORETICAL POWER OUTPUT FROM A FRACTAL SOLAR PANEL

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ABSTRACT

Solar energy is one of the most practical sources of free and convenient renewable energy in the tropics. Capturing this energy and converting it into electricity by a solar or a photovoltaic (PV) cell became commonly pursued in the field of physics, engineering, and renewable energy advocates for decades and in the recent years. The first and second generation solar cells were made in bulk design with the best efficiency ranging from 30 to 40% conversion. Related studies have shown, almost 70% of the losses accounted for thermalization, extraction inefficiencies and non-absorption of solar energy. These known losses can be drawn as a result of properties intrinsic to the material used and geometric design of solar cell. Understanding the wave nature of the solar spectrum and treating it as an electromagnetic wave, it is most viable that the said losses can be recovered using a geometric approach by capturing solar energy at its corresponding wavelengths. It was determined that by employing a fractal Sierpinski's carpet as a PV cell design could theoretically improve the maximum efficiency 3.7 times than the conventional Euclidean PV cell. The effects were supported by related studies in Photonics and Fractal Antennas. Equations for the evaluation of the actual efficiency performance and maximum power point had also been established.

Keywords: fractal, solar panel, solar energy, Sierpinski's carpet, PV cell

Introduction

Solar energy is the most convenient source of renewable energy today and the coming century. The adverse effect of burning conventional hydrocarbons are eminent through global warming and health effects in many forms, including factors aggravating cancer and other illnesses. The enormous volume of fossil fuel was extracted from the ground through the years and with its finite source. Its supply will diminish and by the time it happens, it will become one of the worst crises in the history of man on a worldwide scale.

Remarkably, the sun shines twelve hours a day in the tropics, 100% free and a convenient source. It makes an average 1kW per sq. m of energy available at the surface of the Earth. Eventually, almost every dynamics in nature of weather to primary productivity, nutrient, and other biogeochemical cycles depends on solar irradiance.

It was cited from the Basic Energy Sciences Workshop on Solar Energy Utilization in 2005 that the world's demand for energy would double by the year 2050 and triple by the end of the century. Also mentioned in the same report that the world's current consumption of energy in a year would only be less equal to an hour solar irradiance and yet in 2001, solar energy shares only 0.1% from the rest of other energy resources in which fossil fuel still took the most portion.

There are several divisions of solar energy conversion and technology initiatives to mention: one is the solar fuel, two is solar thermal and three, is the solar electricity which is the most convenient and practical out of the three.

The brief history of the developments in research on solar electricity conversion as shown in Fig. 2 by a (PV) cell started somewhere late of 1975 to 1980 with efficiencies ranging from 2% to 24% (NREL, 2015). Several technologies were already on research for improvement at that time. PV cell technologies like Thin Film, Single junction GaAs, and Crystalline Si Cells pioneered the advances of which the single crystal single junction GaAS had the best conversion efficiency from the rest of that era.

Recently, according to National Research in Energy Laboratory (NREL), a four junction concentrator from Fraunhofer ISE/Soitec, attained the top notch efficiency so far at 46%.

Several issues and difficulties of solar energy conversion are left unsolved and unperfected on PV cell and these areas are the directions of research for several years now.

One of the major issues or limitation of PV cell which is material dependent is band gap. The band gap is the minimum threshold energy that a solar spectrum needs to overcome before conduction and for energy conversion to happen. Fig. 1 shows different semi-conducting materials with the corresponding amount of threshold energy. Band gap eventually, is one of the factors that restrict the conversion efficiency as it consumes some portions of the solar energy.

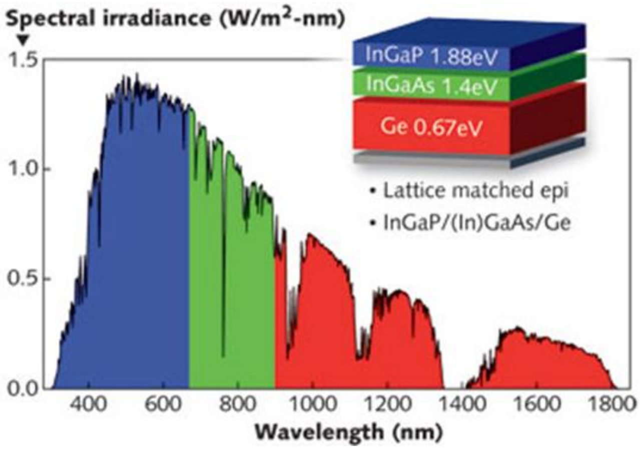

Fig.1 Band gaps and Bandwidth Response *(www.laserfocusworld.com)*

Another issue that hinders conversion efficiency is material bandwidth response. The response of a semi-conductor is limited only to certain portions of the wide bandwidth of the solar spectrum. For example, in Fig. 1, Ge is only capable responding near-infrared bandwidth approximately from 900 to 1800 nanometer wavelengths, InGaAs in the mid portion and InGaP in the region extending to ultraviolet wavelengths. Thus, construction of PV cell of this type is multi-junction and multi-material based. Producing this type of PV cell that captures the wide solar band gap is very expensive. Nathan Lewis, Chairperson of Basic Research Needs for Solar Utilization at US Department of Energy (DOE) quoted that 1 sq. cm of this kind of cell would cost about $40,000, which make this impractical to market and produce in volume.

The subsequent effect of the two issues mentioned above resulted in conversion losses such as thermalization of material and non-absorption of solar energy (Semonin, 2012).

In connection to these PV cell limitations, the research is directed towards the improvement of the solar cell by geometric architecture manipulation using the Fractal Sierpinski carpet. This is also done by establishing equations essential for its actual evaluation.

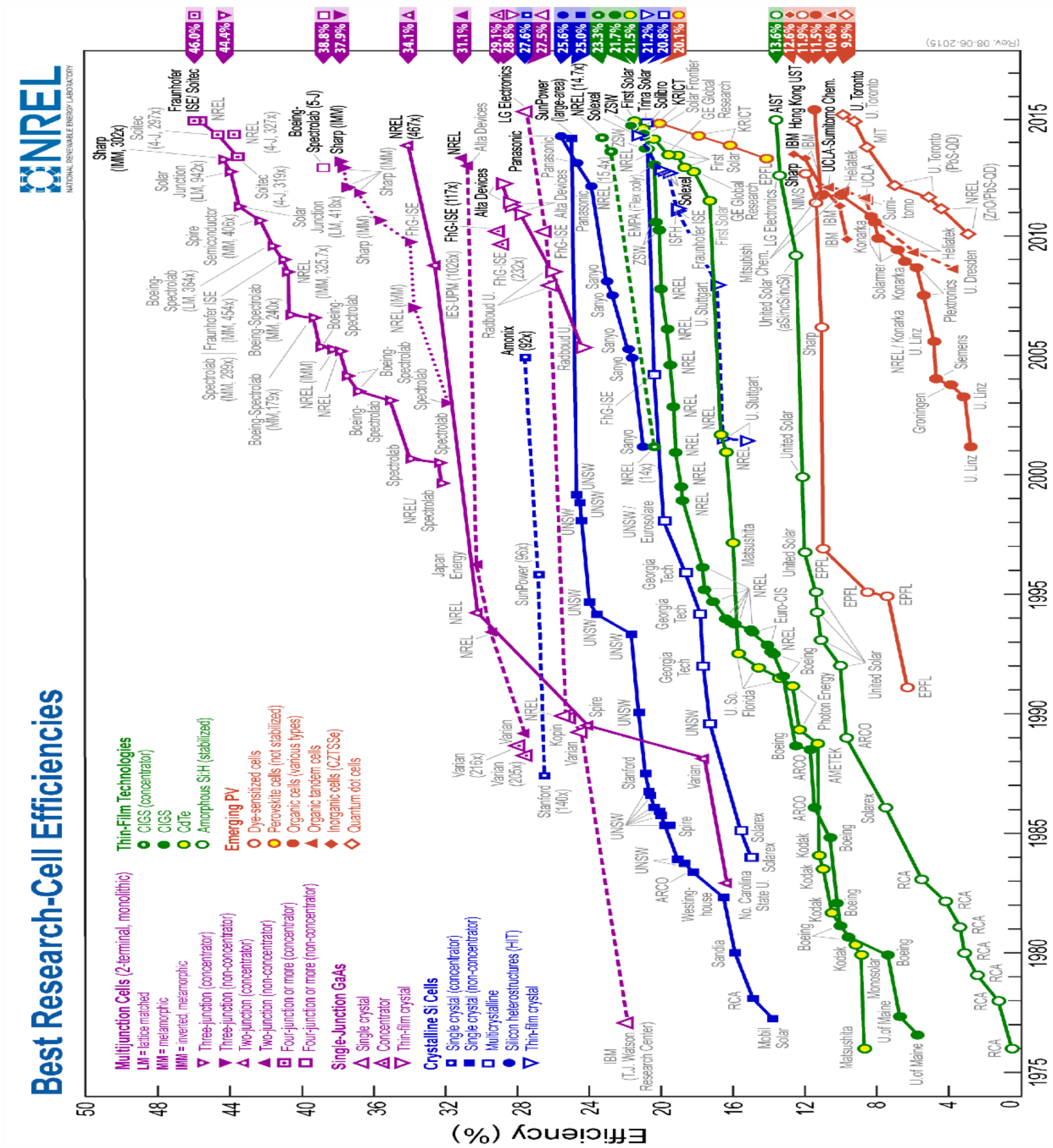

Fig.2 Solar Cell Research history
(courtesy of http://www.nrel.gov/ncpv/images/efficiency_chart.jpg)

Basic Concept and Framework

Solar energy travels to the earth at a speed of 299,792,458 meters/second. The visible light spectrum has a bandwidth of 430 to 790 THz or 390 to 700 nm wavelengths (Starr 2005). At high noon with a cloudless day, the surface of the Earth could receive 1,000 W/m^2 of solar energy (US DOE 2003).

Definition 1. The solar cell efficiency ratio (η) is defined as:

$$\eta = \frac{P_m}{EA} \qquad (1)$$

where Pm = cell power output (watts) at its maximum power point,
E = Solar Irradiation
(W/m^2) and A = surface area of the exposed solar cell (m^2).

The maximum power point Pm, however, can be expressed as the product of the fill factor (FF), the maximum voltage generated at open circuit (V_{oc}) and the maximum current at short circuit (I_{sc}) (Etgar 2013).

$$P_m = FFV_{oc}I_{sc} \qquad (2)$$

This is where short circuit current (I_{sc}) is equal to the current generated by the solar cell directed from n to p side which is in the opposite direction to the generated current from irradiation. This is given in equation 3 (Mahajan 1999).

$$I_{sc} = I_{op} = qAg_{op}(L_p + L_n) \qquad (3)$$

The part $qg_{op}L$ of the above equation is expressed in total charge (Coulomb) per area per time and intrinsic on material property and design. Conversely, short circuit current (I_{sc}) is directly dependent and proportional to the area (A) of the junction exposed to irradiation, diffusion length (L) and generation rate at open circuit (g_{op}).

The open circuit voltage (V_{oc}) alternatively, is expressed in equation 4 below (Mahajan 1999).

$$V_{oc} = \frac{k_B T}{q} \ln\left[\frac{(L_n + L_p)g_{op}}{(L_p / \tau_p)P_n + (L_n / \tau_n)N_p} + 1 \right] \qquad (4)$$

The Fill Factor (FF) on the other hand, is the ratio of the maximum power point (Pm) over the product of the short circuit current (I_{sc}) and the maximum voltage (V_{oc}) at open circuit. In equation 5, this is expressed as a percentage of the actual power generated by the power available at open and short circuits (Etgar 2013)

$$FF = \frac{P_m}{V_{oc} I_{sc}} \tag{5}$$

Conventional solar cell configurations use the usual Euclidean geometry. For instance, in traditional photovoltaic cells, a slab (or wafer) of pure silicon is used to make PV cells. The top of the slab is very thinly diffused with an "n" dopant such as phosphorous. On the base of the slab, a small amount of "p" dopant, typically boron is diffused. Dopants are similar in atomic structure to the primary material. The phosphorous has one more electron in its outer layer than silicon, and the boron has one less. The phosphorous gives the wafer of silicon an excess of free electrons, so it has a negative character. The n-type silicon is not changed since it has an equal number of electrons and protons but some of the electrons are not held tightly to the atoms. They are free to move to different locations within the layer. The boron gives the base of the silicon a positive character. When the n and p-type silicon meet, free electrons from the n-type flow into the p- type for a spit second, then form a barrier to prevent some electrons moving between the two sides. This point of contact is called the p-n junction. When the PV cell is placed under the sun, photons of light strike the electrons in the p-n junction and energize them knocking them free from their atoms. These electrons are attracted to the positive charge in the n-type silicon and repelled by the negative charge in the p-type. Most photon-electron collisions occur in the silicon base.

The usual Euclidean geometry used in the construction of solar panel or PV cells is not necessarily the optimal geometry to be employed to maximize efficiency. For the same energy output Pm, it is possible to configure the solar collectors so that the surface area exposed is smaller. To do this, fractal geometry (Mandelbrot, 1982) provides a convenient platform for this purpose.

Fractals are geometric figures characterized by self-similarity at various scales. The repetition of patterns at all scales gives fractals the appearance of chaos. The fractal dimension (λ) is an indication of how much space is filled by the geometric figure. Thus, for fractals drawn in flat two-dimensional surfaces, the fractal dimension cannot exceed two, but will be greater than 0 (point), i.e. $0 < \lambda \leq 2$.

Definition 2. The box-counting fractal dimension (λ) is given by:

$$\lambda = \frac{\log m}{\log r} \qquad (6)$$

where m = number of copies at the first iteration, r = scale.

The simplest fractal that can be constructed from a unit interval I = [0, 1] is the Cantor set or fractal dust. Royden (1980) provides the iterative process of constructing C as follows: Divide the unit interval into three (3) equal parts and remove the middle third. From the two (2) fragments, repeat the process ad infinitum. The fractal dimension of the Canter set is:

$$\lambda = \frac{\log m}{\log r} = \frac{\log 2}{\log 3} \square\ 0.67$$

Let C be the Cantor set and μ(.) be the usual Lebesgue measure on I. Then μ(I) = 1 while μ© = 0. The Cantor set C, however, has the same number of points as I and is also uncountable (Royden, 1980). This remarkable fact is the basis for the modern application of fractals in technology.

For regular Euclidean shapes, the ratio of the perimeter P to the square root of area A is constant k regardless of the size of the shape:

$$k = \frac{P}{\sqrt{A}} \qquad (7)$$

For a square, k = 4; for a circle $K = 2\sqrt{\pi}$. For areas bounded by fractal curves, where the length of the perimeter diverges as the subdivisions and become smaller and smaller, the relationship is:

$$k = \frac{P^{1/\lambda}}{\sqrt{A}} \qquad (8)$$

Where P and A are the measured perimeter and area using scale λ, respectively (Addison, 1997).

Results and Discussion

A single unit of solar collector in the shape of a square is fractalized into a Sierpinski's carpet as shown in the figure below:

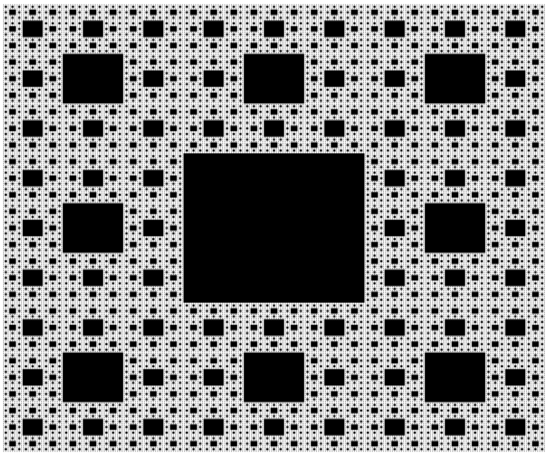

Fig.3 Sierpinski Solar Collector *(courtesy of http://paulbourke.net/fractals/carpet/hadamard.gif)*

The Sierpinski carpet is a plane fractal first described by Waclaw Sierpinski in 1916. The construction of the Sierpinski carpet begins with a square. The square is cut into nine (9) congruent subsquares in a 3x3 grid and the central sub-square is removed. The same procedure is then applied recursively to the remaining 8 sub-square and infinitum.

Theorem 1. The area of the Sierpinski carpet tends to zero with respect to the standard Lebesgue measure.

Proof: Let A; be the area of the carpet at iteration i. Then,

$$A_{(i+1)} = \left(\frac{8}{9}\right) A_i \qquad (9)$$

hence,

$$A_i = \left(\frac{8}{9}\right)^i \to 0 \qquad \text{as } i \to \infty$$

Theorem 2. The fractal dimension of the Sierpinski carpet is $\lambda = 1.8928$.

Proof: Since m=8 and r=3, then $\lambda = \frac{\log 8}{\log 3} \simeq 1.8928$.

Theorem 3. The area of the Sierpinski at a finite iteration i=N is less than the unit square.

Proof: From Addison (1997),

$$\lambda = \frac{\log 8}{\log 3} = 1.8928$$

$$A = \frac{P^{2/\lambda}}{k^2} \quad \text{where } k = 4 \text{ for a square.}$$

Hence, $A = \frac{4}{4^2}^{2/\lambda}$ Since λ=1.8928 .

By utilizing fractal architecture in the design of solar collectors, it is possible to increase efficiency by more than 100% (see Definition 1) by decreasing the term A_c in the denominator. However, the numerator P_m which is the power output might also be affected by the decrease area.

Depending on the design of the PV, the modules can produce electricity from a range of frequencies of light (ultraviolet, infrared, and low light) but the conventional geometric design cannot cover the entire solar range. Hence, much of the incident sunlight energy is wasted by solar modules. They can give far higher efficiencies if illuminated with monochromatic light. A design concept is to split the light into different wavelength ranges and directs beams onto different cells turned to those ranges. This increases efficiency by 50%. Developments in fractal antennas demonstrate that this could be possible in solar energy technology (Hodlmayr, 2007).

With a different geometry, namely fractal geometry, the new, highly convoluted irregular shapes of fractals; the solar collectors allow graphical scaling to a degree where the multi-frequency operation is possible with very small sizes and multiple operating bands (Hodlmayr, 2007). The better the curve fills up a surface, the better for the multi-frequency solar collector. This is the principle of lacunarity.

Theorem 4. The principle of lacunarity operates in fractal Sierpinski solar collectors.

Proof: Since the space-filling property of the Sierpinski carpet is high with $\lambda = 1.8928$, it follows that it will have higher multi-frequency solar energy collection property. Hence, the principle of lacunarity applies.

Another consequence of the fractal shape is the existence of a multitude of "places" along the silicon base where charges are strongly accelerated, hence, producing electricity.

From Theorem 4, we deduce the P_m under the Sierpinski architecture.

Main Result. The efficiency of a solar collector designed under the usual Euclidean Geometry is less than the efficiency of a solar collector using the fractal Sierpinski architecture.

Proof: Combine Theorem 3 and Theorem 4.

Note that $A_{Sierpinski} = 0.27$ so that the efficiency is theoretically raised by about 370%.

The computed theoretical efficiency means the maximum limit that a fractal Sierpinski carpet could approach, 3.7 times than the Euclidean PV cell.

The value does not represent reality as the computation is based on the ideal condition that material used undergoes perfect absorption and no entropy production, a mathematical representation influenced by fractal arrangement. Fractals extend towards infinity, thus it is expected to get values out of bound.

The significance of the computed efficiency is that it multiplies Euclidean PV maximum limit of efficiency of 100% to 3.7 times to some extent and this fact alone is worth investigating and an indicator of improvement.

Projected Power Output of Conventional Solar Panel

On the other hand, looking at the maximum power output of the Euclidean designed PV cell by combining equations 3 and 4 previously discussed, the expression below for maximum power point can be derived as

$$P_m = FFAg_{op}k_BT(L_p + L_n)\ln\left[\frac{(L_p + L_n)g_{op}}{(L_p/\tau_p)P_n + (L_n/\tau_n)N_p} + 1\right] \quad (10)$$

Equation 10 is obtained by the product of the Fill Factor FF, open circuit voltage V_{oc} and short circuit current I_{sc} of the PV cell exposed to irradiation as given by equations 3 and 4.

The P_m is directly proportional to the area A of the junction perpendicular to the light rays, g_{op} generation rates of holes and electrons of the material at open circuit, L length of the diffusion region of the p-side and the n-side, the absolute operating temperature T and lifetime of electrons and holes τ in the PV cell. The constant k_B is the Boltzman constant and the term P_n and N_p are the concentration of holes and electrons in the p and n type materials.

There is a significant amount of losses in a typical conventional PV Cell. Semonin (2012) cited in Fig. 4 that these losses accounted about 57% of the total available solar spectrum, rejected as thermalization, extraction inefficiencies and unabsorbed solar rays for a single-junction bulk crystalline semi-conductor.

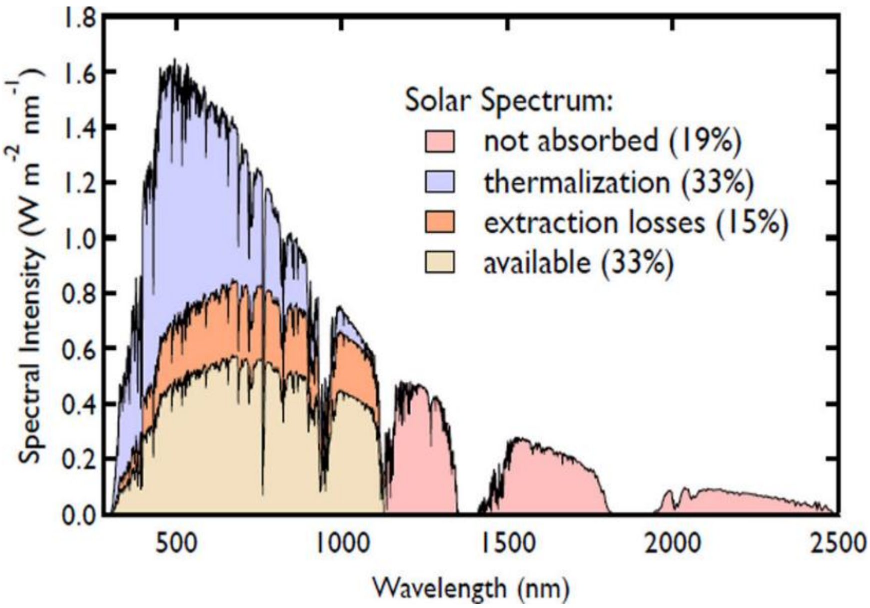

Figure 4 Accounting of energy conversion of a bulk crystalline PV Cell (*http://spie.org/Images/Graphics/Newsroom/Imported-2012/004146/004146_10_fig1.jpg)*

Projected Power Output of Theoretical (Sierpinski Carpet) Fractal Solar Panel

The approach of fractal PV cell is to unite the solar spectrum bandwidths with the sizes or areas of PV junctions capturing all the wavelengths. This is simply treating light as an electromagnetic wave and handling it similarly as radiation and its absorption similar to an antenna. Studies of Baharwadj & Novotny (2009) cited in their work on photonics and optical antennas, that absorption, radiation, and control of the electromagnetic light spectrum is more efficient on antenna sizes with their corresponding wavelengths. The same fundamental concepts of the power receiving antenna as mentioned by Freeman (2011) in his lecture on the topic Signals and Systems.

The maximum power point of a fractal solar cell could be expressed as the sum of its individual aggregates from i to n generations. Simple manipulation of equation 8 to calculate for the maximum power point P_{mf} for a Sierpinski Carpet Solar Cell is shown below.

$$\sum_{i=1}^{n} \frac{P_{mi}}{N^{(i-1)} d^{2i}} = \sum_{i=1}^{n} \eta_i E \quad (11)$$

$$Af = \sum_{i=1}^{n} N^{(i-1)} \delta d^{2i} \quad (12)$$

Where:

N – base number of fractal areas (N=8 for Siepinski Carpet)

$A_i = \delta d^{2i}$, Area size of aggregate at corresponding i generation (d = 1/3 & $\delta = 1$ for Sierpinski Carpet)

A_f - Total area of aggregated solar junction from $i = 1$ to n generation

P_{mi} – Maximum Power Point at corresponding aggregate area of i generation of the junction

η_i – Solar Cell Efficiency of junction area at i generation

E – Solar Irradiance A.M 1.5 G (1,000 W/m^2)

Equation 11 means that the specific solar efficiency of the aggregated solar cell at i is at its maximum when the ratio of the maximum power point of generation i and the sum of all the areas of the aggregate at that generation would approach the value of unity in the solar Irradiation E .

As the dimension d of the squared region approaches 390nm-the least wavelength of the visible spectrum in equation 12, it is expected that there should be an increased in photon energy absorption in photovoltaic material. Conversely, it is anticipated that the maximum power point per junction area at generation i is also at its optimum in these nanometer dimensions. This is because of the higher g_{op} of the PV cell and longer τ lifetimes of hole-electron changes depicted in equation 10 and incoherence to the result and observation of Hägglund (2008).

Hägglund (2008) illustrated that there are three subsequent effects in Fig. 4 on a light absorber in the nanometer dimensions that would explain the increase of absorption or conversion efficiency.

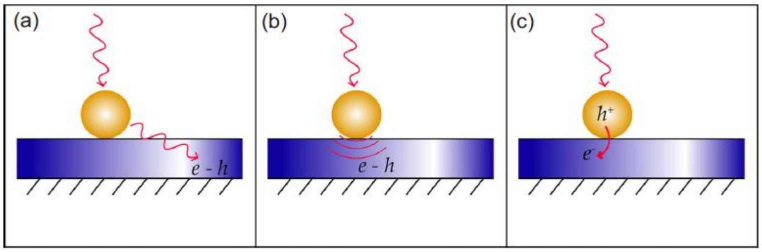

Figure 5. Different types of antenna effects in photovoltaics. (a) Far-field scattering, leading to a prolonged optical path.(b) Near-field scattering causing locally increased absorption, and (c) direct injection of photoexcited carriers into the semiconductor (www.osapublishing.org).

These are the phenomena that could occur at nanometer scale dimensions as illustrated in the Fig. 5 (a) far-field scattering of photons which effect longer lifetimes of hole-electron charges, (b) near-field scattering would increase hole-electron generation rates g_{op} and lastly, (c) direct injection which is a direct conversion of high-intensity photons to electron-hole energy.

The P_{mf} of a fractal solar panel could be expressed as the total sum of the maximum power points of the aggregates at $i = 1$ to n generations in equation 13. Likewise, its solar efficiency η_f is the total efficiencies of the aggregates from $i = 1$ to n generation expressed in equation 14.

$$P_{mf} = \sum_{i=1}^{n} P_{mi} \tag{13}$$

$$\eta_f = \frac{\sum_{i=1}^{n} \eta_i}{n} \tag{14}$$

However, we note that the total solar efficiency η_f is a straight forward independent efficiency of aggregate areas from $i=1$ to n generations. Basically, this does not include yet, the influence of the collective effects of the fractal arrangement on the PV cell. With this, a fractal coefficient that denotes the influence of fractal arrangement on solar efficiency is introduced and defined as γ_f . Its possible values could range +/- proportions of unity.

Finally, the overall solar efficiency of a fractal solar panel η_F could be expressed in equation 15 and 16 below relating to P_{mf} , the total area of PV cell junction A_f and E .

$$P_{mf} = \eta_F E A_f \tag{15}$$

$$\eta_F = (1+\gamma_f)\eta_f \tag{16}$$

Conclusion

Theoretical results appear to support fractal architecture in the design of a solar panel. The fractal architecture maximizes the absorption of solar spectra not otherwise captured in a Euclidean architecture of the solar panel. These theoretical results, however, are extremely challenging to verify in a laboratory setting as they require measurements at the nano-scale.

Fractal geometries could be the solution to eliminate multi-junction and multi-material PV cell, thus lowering cost of production and complexities. Also, higher efficiencies could be achieved further as a material selection of the lowest band gap could be made possible by just employing fractal arrangement at corresponding solar spectrum wavelengths.

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